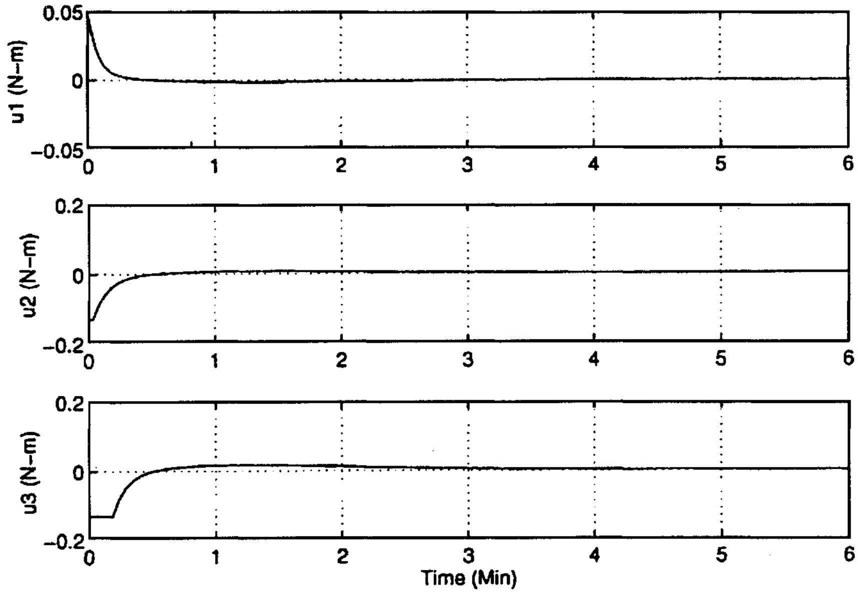

Fig. 3 Plot of applied control torques.

and the constant λ is set to $-0.015\ s^{-1}$. The parameter ε in the saturation controller is set to 0.01. Also, the control torques are limited to 1.0 N-m. A plot of the closed-loop modified Rodrigues parameters is shown in Fig. 1. Also, plots of the angular velocity trajectories and applied control torques are shown in Figs. 2 and 3, respectively. Using Eq. (3), the rotation for the initial conditions in Eq. (21) is approximately 193 deg. Therefore, converting the modified Rodrigues parameters shown in Fig. 1 to quaternions[6] reveals that the scalar (fourth) quaternion crosses the zero point. Therefore, the Gibbs vector control formulation in Ref. 3 becomes singular but is easily handled by the modified Rodrigues parameter control formulation.

Conclusions

In this Note, a sliding mode controller was developed for attitude pointing using the modified Rodrigues parameters. The modified Rodriques parameters represent a minimal parameterization with a singularity at 360 deg. These parameters avoid the normalization constraint associated with the quaternion parameterization and allow for rotations of greater than 180 deg for which the Gibbs vector parameterization becomes singular. Simulation results indicate that the new algorithm was able to accurately control the attitude of a spacecraft for large-angle maneuvers.

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Application of a Learning Control Algorithm to a Rotor Blade Tab

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Introduction

THE suppression of vibration is of continuing interest in rotorcraft technology. Because of periodic excitation inherent in forward flight, a rotorcraft is subject to varying dynamic and aerodynamic loads. These variable loads act on the main rotor, and attempts to alleviate them provide the motivation for many studies of various rotor design concepts and of the application of active control either to rotor pitch (higher harmonic control and individual blade control concepts)[1] or to specially designed additional devices.[2]

A tab mounted at the blade trailing edge has generated much interest recently for additional rotor control. This has arisen from prospects of providing the driving mechanism for the tabs through smart structure technology.[3] Several analytical and experimental studies have been carried out to obtain insight into different aspects of its application.[4] Tabs driven by piezoelectric benders were tested experimentally on a rotor model in hover.[5]

Physical phenomena involved in applications of the "smart tab" are aeroelastic [i.e., including both dynamic (inertia and elastic loads)] and aerodynamic phenomena.

To achieve the required goal a proper control strategy should be applied to the system. Consideration has been given to open loop in Ref. 6, and control algorithms of mainly optimal linear quadratic

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or linear Gaussian type in the frequency domain have been utilized in Ref. 7. Also some heuristic approaches[8] in the time domain have been tested.

The objective of this study is to investigate the possibility of the application of a time domain learning algorithm for controlling a tab mounted at the trailing edge of a blade to diminish the rotor vibration level.

The algorithm is designed to influence the periodic motion of the system. The vibration reduction considered here is expressed as the requirement for the blade to perform a given required motion. The capability of influencing the blade motion allows improvement of blade aeroelastic behavior and alleviation of blade loads at the blade root. This allows the reduction of unwanted components or harmonics of loads transferred to the fuselage structure.

Background of the Control Method

In rotorcraft aeroservoelastic problems, the plant to be controlled is periodic with respect to time. There have been attempts to develop control algorithms for such types of plant in rotorcraft research.[9] Similar activity has been performed in the robotics area, although the plant considered in this field seems to be more easily handled.

The control algorithm applied in the present study is a modification of that developed in Refs. 10 and 11. The term "learning control" was introduced there by way of explaining that "information from the previous trial of the plant dynamics and the tracking error at each time step is reflected in the next trial."

The discrete, linear system periodic with respect to time with scalar control $u(k)$ is considered:

$$x(k+1) = C(k)x(k) + D(k)u(k) + d(k) \tag{1}$$

where $k = 1, 2, \ldots, M$ are the numbers of time steps.

State matrix $C(k)$ and control matrix $D(k)$ are periodic with respect to time, i.e., for all k,

$$\begin{aligned} C_{i+1}(k) &= [C_{mn}(k)]_{i+1} = C_i(k) \\ D_{i+1}(k) &= [D_{m1}(k)]_{i+1} = D_i(k), \qquad m, n = 1, \ldots, N \end{aligned} \tag{2}$$

In the preceding, the subscript i describes the number of the period.

The periodic disturbance $d(k) = [d_n]$, $n = 1, \ldots, N$, is bounded and for all k, and i fulfills the condition

$$\|d_{i+1}(k) - d_i(k)\| \le \varepsilon_d \tag{3}$$

where ε_d is a prescribed constant.

The learning problem is stated as the requirement that the state vector $x_d(k)$ is a realizable, periodic trajectory. The sequence of control applied $u_i(k)$, $i = 1, 2, \ldots,$ should provide that, starting from some period of time, the system trajectory $x_i(k)$ will satisfy the condition

$$\|x(k) - x_d(k)\| \le \varepsilon_0 \tag{4}$$

where ε_0 is the assumed tolerance bound.

It was proved in Ref. 9 that the control defined as

$$\begin{aligned} u_{i+1}(k) &= u_i(k) + \lambda\big[\hat{D}_i^+(k) : -\hat{D}_i^+(k)\hat{C}_i(k)\big] \\ &\quad \times [e_i(k+1) : e_i(k)]^T \\ e_i(k) &= x_d(k) - x_i(k) \end{aligned} \tag{5}$$

fulfills the learning condition if, for initial error $e_i(0) = 0$, the estimate of matrix $\hat{D}(k)$ satisfies the condition

$$\left|1 - \lambda\hat{D}_i^+(k)D(k)\right| < 1 \tag{6}$$

In the preceding the superscript + denotes the generalized matrix inversion $[A^+ = (A^TA)^{-1}A^T]$ and the symbol ^ denotes the estimated value. The symbol : indicates combining two matrices or vectors into a single partitioned matrix or vector.

If the external disturbance is periodic, then

$$\|e_i(k)\| \to 0, \qquad \text{for} \qquad i \to \infty \tag{7}$$

These expressions form the basis for application of this algorithm to a nonlinear, continuous system.

Application to a Nonlinear, Continuous System

The mathematical model of a helicopter rotor blade can be expressed as a nonlinear system of ordinary differential equations periodic with respect to time, with scalar control $u(t)$ corresponding to the angle of deflection of the trailing-edge tab

$$\dot{x} = f(t, x, u) \tag{8}$$

where the symbol · denotes differentiation with respect to time t.

For assumed nominal tab control $u_d(t)$ the desired periodic solution for this equation is $x_d(t)$.

The system (8) is linearized about $x_d(t)$:

$$\dot{x} = A(t)x + B(t)u(t) + R[x, x_d, u(t), u_d(t), t] \tag{9}$$

The matrices $A(t)$ and $B(t)$ in Eq. (9) are defined as

$$A = [A_{ij}] = \left[\frac{\partial f_i}{\partial x_j}\right]_{x_d, u_d}, \qquad B = [B_i] = \left[\frac{\partial f_i}{\partial u}\right]_{x_d, u_d} \tag{10}$$

and the quantity $R[x(t), x_d(t), u(t), u_d(t), t]$ contains the higher-order terms.

Approximating the time derivative by the forward finite difference

$$\dot{x} = \frac{x(t + \Delta t) - x(t)}{\Delta t} \tag{11}$$

and inserting it into the linearized equation (9) transfers it to the discrete time domain

$$\begin{aligned} x(t + \Delta t) &= [I + A(t)\Delta t]x + B(t)\Delta t u(t) \\ &\quad + R[x, x_d, u(t), u_d(t), t]\Delta t \end{aligned} \tag{12}$$

and by substitution,

$$\begin{aligned} t = k\Delta t, \qquad C(k) &= [I + A(k\Delta t)\Delta t] \\ D(k) &= B(k\Delta t)\Delta t \\ d(k) &= R[x(k\Delta t), x_d(k\Delta t), u(k\Delta t), u_d(k\Delta t), k\Delta t] \end{aligned} \tag{13}$$

Equation (12) can be reformulated to the form of Eq. (1).

The proposed application of this control algorithm to the nonlinear case consists of the following steps: 1) dividing the time period into M steps by prescribing the points of time $t_k = k\Delta t$, $k = 1, 2, \ldots, M$, and calculating at these points: a) the desired solution $x_d(k)$; b) matrices $A(k)$ and $B(k)$ of the linearized, continuous system (10); c) matrices $C(k)$ and $D(k)$ according to Eq. (13); and d) the gain matrix $G(k)$ according to the formula

$$G(k) = \big[\hat{D}_i^+(k) : -\hat{D}_i^+(k)\hat{C}_i(k)\big]^T \tag{14}$$

2) assuming the value of λ (as the theory gives no indication for its selection); and 3) starting from $e_0(k) = 0$ the system is controlled in each period of time according to the formulas

$$\begin{aligned} u_{i+1}(k) &= u_i(k) + \lambda G(k) \times \big[e_i(k+1) : e_i^T(k)\big]^T \\ e_i(k) &= x_d(k) - x_i(k) \end{aligned} \tag{15}$$

In this approach the higher-order terms $R(t, x, u)$ rejected in linearization are treated as a disturbance vector $d(k)$ and matrices $C(k)$ and $D(k)$ as estimates of the matrices of the discrete periodic system.

Application of the Algorithm to the Blade Tab

In this study the algorithm is used to obtain the required motion of a helicopter rotor blade. This task can be considered as, for instance, an attempt to suppress the prescribed harmonics of the blade motion.

During numerical simulation the sequence of calculation is as follows:

1) Describe the blade required motion. For this purpose a) the nonlinear system (8) of blade equations of motion is solved for a prescribed number of rotor revolutions. The blade motion during the last rotation is taken to be the blade steady motion $x(k)$. b) A Fourier analysis of the steady motion $x(k)$ is performed.

Table 1 Blade data

Rotor angular velocity Ω, rad/s	34.0
Air density, kg/m^2	1.226
Blade chord (aerofoil + tab), m	0.395
Rotor radius, m	6.40
Blade mass, kg	57.2
Blade length, m	5.61
Natural frequency of twist, 1/Ω	6.29
Damping of aerofoil twist, % crit	0.01
Linear twist of the blade, deg	from 4.3 to −2.2

The blade required motion $x_d(t)$ is reconstructed using the selected Fourier series coefficients from step b.

2) Calculate the control gain matrix according to Eq. (14) by linearizing the blade equations about the reconstructed steady motion and assumed initial control.

3) Apply the learning algorithm to simulate controlled blade motion for the assumed number of blade rotations.

Blade Model

For the numerical simulation of helicopter rotor blade motion, a well-tried and tested computer model of an individual blade[12] is utilized, adapted for inclusion of a trailing-edge tab. The motion of a single rotor blade of a helicopter in a steady flight is studied. The angular velocity of the rotor shaft is constant.

The blade has a straight elastic axis and is pretwisted about it. The blade stiffness loads are obtained from a Houbolt–Brooks model. The blade cross sections have symmetry of elastic properties about a chord and there is no section warping. Viscous structural damping of blade deformations is included. The aerodynamic loads on the blade are calculated from a two-dimensional, quasisteady, nonlinear model based on a table lookup procedure, supplemented by a procedure for calculating loads on the aerofoil with a tab described in Ref. 13. The induced velocity is calculated from the Glauert formula.

The blade model allows for different hub arrangements and blade properties. The blade deflections are discretized using free vibration rotating modes.

A tab would most likely influence the blade twisting moment, and so this was the reason for selecting this blade degree of freedom for the current investigation. To test the control algorithm, a hingeless blade rigid in bending and elastic in torsion was selected from the available blade models. The base configuration comprises the blade deformable in twist attached to the shaft via a stiff element. It can be controlled in pitch about a feathering bearing. One rotating twist mode of deflection is assumed. Numerical results are obtained using as the base data that approximates to the Westland Lynx blade.[14] The main values of blade parameters are given in Table 1.

For numerical integration of the equations of motion, Gear's algorithm is used, which allows for solution of "stiff equations."

The rotor is in wind-tunnel state. It has fixed collective pitch of 10 deg and no cyclic pitch, and the shaft angle is perpendicular to the flow velocity. The flow velocity, expressed as rotor advance ratio, varies from 0 to 0.35 in 0.05 intervals.

Numerical Simulation

The numerical study raises the following aspects associated with the proposed control algorithm: 1) application of linear methodology to the nonlinear case, 2) simple form of continuous system discretization to the time domain, 3) no rational indications for choosing the parameter λ, and 4) rotor blade aeroelastic behavior that influences controllability of the system. Insight into these aspects of the control can be gained by numerical simulation.

The effectiveness of the control strategy depends both on the plant properties and the control algorithm. As one of the assumptions of the method comprises controllability of the system, the tab mounted at the blade should produce aerodynamic loads sufficient to influence blade motion. The magnitude of these loads depends on the blade dynamic properties, rotor speed, and the helicopter flight velocity.

In the reported approach the difference between the linear and nonlinear case is a contribution to the vector of disturbances $\boldsymbol{R}$ in Eq. (9), which is to be removed by the control algorithm. The plant consists of a nonlinear aeroelastic system. Usually, in practice, the process of system identification is based on the assumption of linearity about the periodic steady state. Thus, there is no problem in identifying matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ in linear equation (10) and then matrices $\boldsymbol{C}$ and $\boldsymbol{D}$ in Eq. (1).

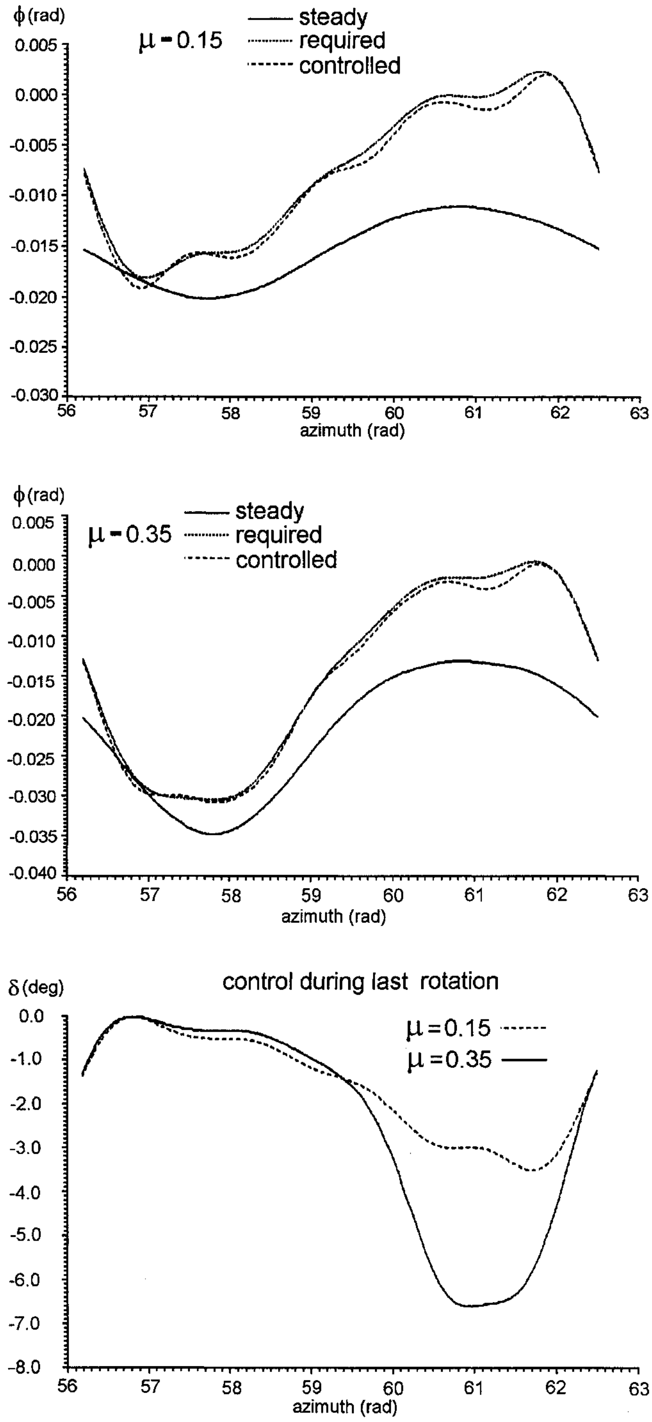

Fig. 1 Required motion composed of selected harmonics.

The study is aimed at validating the effectiveness of the algorithm when applied to an aeroelastic model, and the real-time application is not considered. The latter is discussed in Refs. 10 and 11.

The first result of numerical simulation indicates that, because of the high fundamental torsional frequency of the blade, a tab of chord $0.1c$ needs to extend from the initial inner value of 23.3 to 95% of the blade span.

Another important factor for control efficiency is the control constant λ, which can be adjusted by trial and error. If too large, the control is too aggressive, and if too small, the learning process is slowed. In the case considered, the smallest value of λ that was found to be effective was 0.05.

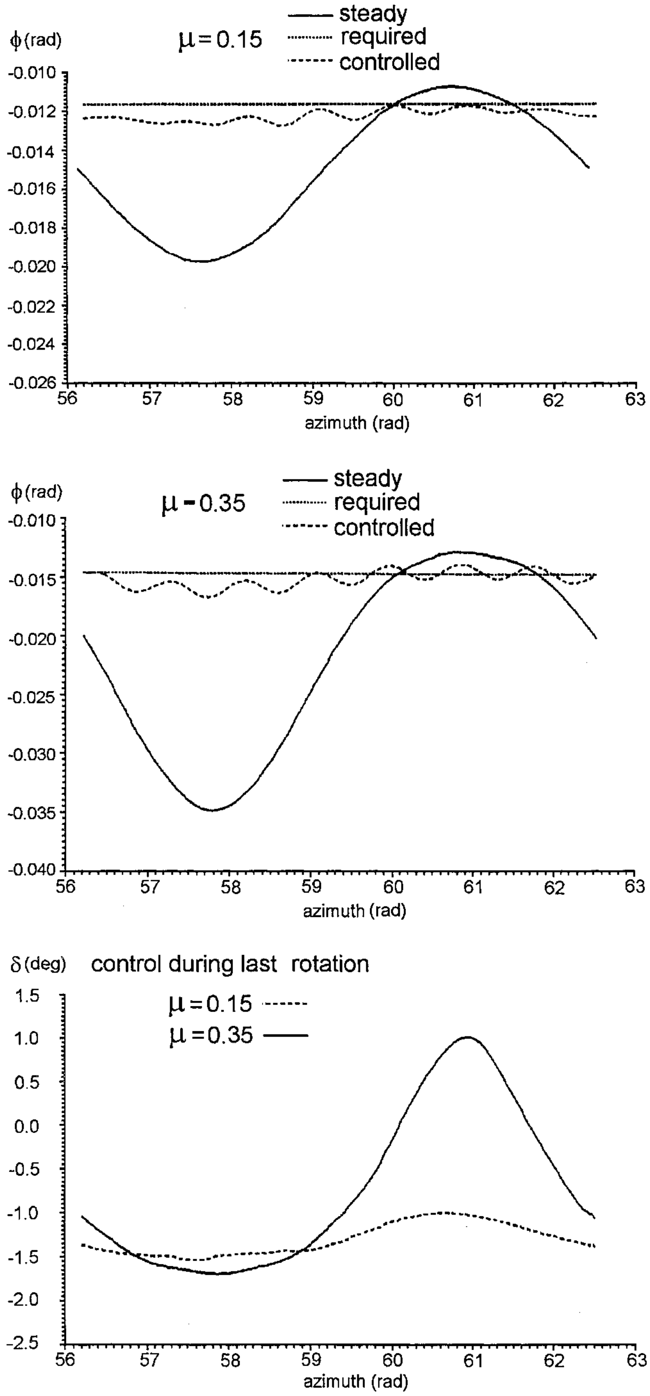

Fig. 2 Required motion of constant values.

For the chosen tab chord and control parameters, the sample results of blade control are given in the figures for helicopter advance ratios of 0.15 and 0.35. These show the motion of the nonlinear system after 10 rotations (which is regarded as the blade steady motion), the required motion for the case considered, and the controlled motion after 10 rotations of the algorithm being applied.

The blade steady motion was obtained by integration of nonlinear equations of motion. Numerical experiments showed that 10 rotations are required to achieve sensibly steady blade motion, and so this number of rotations was chosen for control activity, which was also found to be sufficient to achieve control efficiency.

Two cases of blade motion are considered. Both of these cases are aimed at demonstrating the effectiveness of the control algorithm; it is not intended that they represent real flight situations.

In the first case shown in Fig. 1, the required motion is reconstructed from the Fourier coefficients of steady motion up to the seventh order but without the third and fourth harmonics. In the second case, Fig. 2, the constant component of steady motion is forced by the control algorithm.

In both cases the control algorithm proved to be effective, driving the blade twist to the vicinity of required motion. The required tab deflection is within acceptable limits, although the time dependence of its value varies with the type of the motion required.

The maximum tab deflection needed to achieve the control objectives is larger in the first case than in the second. This can be attributed to the fact that in case 1 the blade is required to follow a prescribed periodic motion, whereas in case 2 the disturbances are rejected and constant deflection of the blade is required.

Conclusions

A learning control algorithm has been modified and applied to the nonlinear periodic system that models a helicopter rotor blade. The numerical simulations have shown that with an appropriate tab length and selected control constant, the blade twist angle can be influenced in such a way that the blade follows the required motion. This demonstrates the efficiency of the control strategy being applied to a periodic, nonlinear system and the efficiency of the control algorithm in the aeroservoelastic case, allowing that the tab size is adequate to influence the blade motion.

The somewhat impractical tab spanwise length needed to fulfill this task suggests that mass and stiffness tailoring of the blade should be a necessary follow-up exercise to obtain effective control within acceptable design parameters.

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